Aspects of Quantitative Program Verification

Kick-off Meeting

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UNIVERSITÄT
DES
SAARLANDES

WS 22/23, November 14, Universität des Saarlandes, Germany

Part I

Organizational Matters

Attendance Check

Please stand by...

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Deliverables:

- Outline + 1 (draft) page of main part of the report
- Final report
- Presentation

Outline + 1-Pager

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- Helps *you* to sort your thoughts, tell a coherent story;
  helps *me* to see whether you're on track and give you early feedback

The Report

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- You did an excellent job if your report is more comprehensible than the original paper!

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- Language: English or German (but you'll find that English is easier)
- I expect correct grammar and spelling
  - ≥ 10 gross errors per page is unacceptable and causes me to discontinue reading your outline / (preliminary) report

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- You did an excellent job if everybody understood what you were talking about!

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- 30 minutes presentation + 10 minutes Q&A
- (hopefully) in-person
- Dates: 1 or 2 days in beginning of March 2023 (tentative, TBA on website)
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Missing any non-optional deadline causes immediate failure of the seminar.

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19.11.–20.11. All of us: Have a good weekend!

from 21.11. You: Start working the seminar

Part II

Topics

Generalized Hoare Logics and Predicate Transformers

Hoare Logics and Predicate Transformers

Predicate G, F: States $\to \{\text{true}, \text{false}\}$

Hoare Logics and Predicate Transformers

Predicate G, F: States $\to$ {true, false}

Hoare triple $\langle\, G \,\rangle\ C\ \langle\, F \,\rangle$

- Executing program C on initial state $\sigma \models G$
  terminates in a final state $\tau \models F$

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Weakest precondition wp $[\![C]\!]\ (F)$

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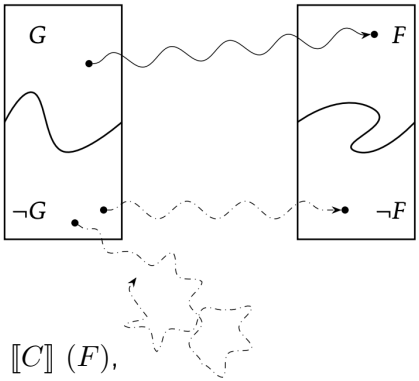

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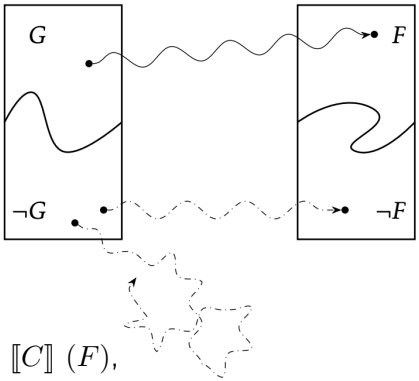

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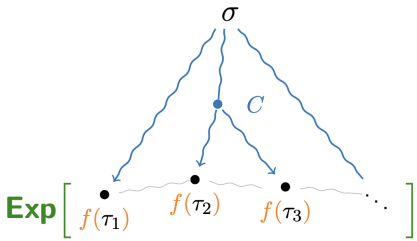
Generalized Hoare Logics and Predicate Transformers

- Replace predicates by objects that map to a more general domain than {true, false}, for instance $\mathbb{R}$.

Generalized Hoare Logics & Predicate Transformers

Quantity f : States $\to \mathbb{R}^{\infty}_{\geq 0}$



Example: Weakest preexpectation wp $[\![C]\!]\,(f)$

- wp $[\![C]\!]\,(f)$: States $\to \mathbb{R}^{\infty}_{\geq 0}$ maps initial state σ to the expected value of f after executing C on σ

Generalized Hoare Logics & Predicate Transformers (Topics)

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- *Klinkenberg et al.* Generating Functions for Probabilistic Programs **(BA)**

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- *Klinkenberg et al.* Generating Functions for Probabilistic Programs **(BA)**
- *McIver & Morgan.* Correctness by Construction for Probabilistic Programs **(BA)**

Incorrectness Reasoning

Incorrectness Logic and Predicate Transformers

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Generalized Predicate Transformers

- Again, replace predicates by objects that map to a more general domain

Incorrectness Reasoning (Topics)

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- *O'Hearn.* Incorrectness Logic. **(BA)**

Incorrectness Reasoning (Topics)

- *O'Hearn.* Incorrectness Logic. **(BA)**
- *Zhang & Kaminski.* Quantitative Strongest Post — A Calculus for Reasoning about the Flow of Quantitative Information **(BA)**

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  - Weakest precondition wp $[\![\texttt{while}\,(\,\varphi\,)\,\{\,C\,\}]\!]\,(\psi) = \text{lfp}\ \Phi$
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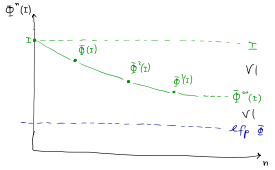

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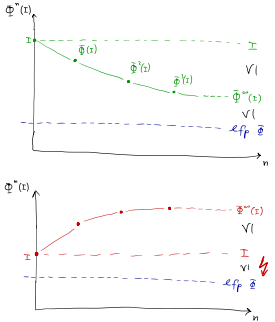

Lower Bounds on Least Fixed Points (Topics)

- What about lower bounds?

$$I \preceq \Phi(I) \quad \not\Rightarrow \quad I \preceq \mathsf{lfp}\ \Phi$$

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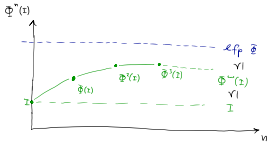

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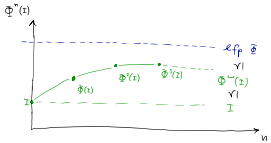

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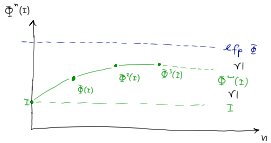

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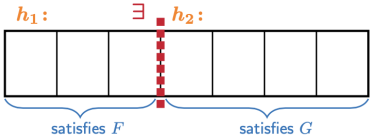

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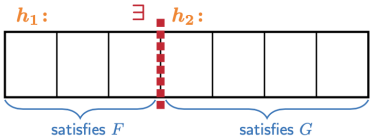
  - Enables local reasoning!



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- *Barthe, Hsu, & Liao.* A Probabilistic Separation Logic
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Neural Networks

Tropical Geometry of Neural Networks

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- *Zhang, Naitzat, & Lim.* Tropical Geometry of Neural Networks

Part III

Final Thoughts

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